Exercises

Spectrum of sinusoidal signals

The exercises are split into the following three categories:

- The exercises in the table below are Mandatory. These exercises must be prepared in such a way that they can be presented during the compulsory practice hours.
- Pencast [P] exercises, from which a complete work-out is available in a pencast video.
- The resulting exercises are available for additional training.

Subject		Exercise
Spectrum	M1	Ex.1
	M2	Ex.2
	M3	Ex.3
	M4	Ex.5

Exercise 1
A signal composed of sinusoids is given by the equation

$$x(t) = 3\cos(50\pi t - \pi/8) - 5\cos(150\pi t + \pi/6)$$

Make a plot of the spectrum of this signal. Plot on the horizontal axis the frequency in [Hz] and indicate for each frequency of the signal a bar indicating the complex amplitude (magintude and phase)of each frequency component.

Exercise 2
Fig. 1 shows several signals along with their corresponding spectra. However, they are in a random order. For each spectrum plot (a)—(e), determine the correct signal (1)—(5).

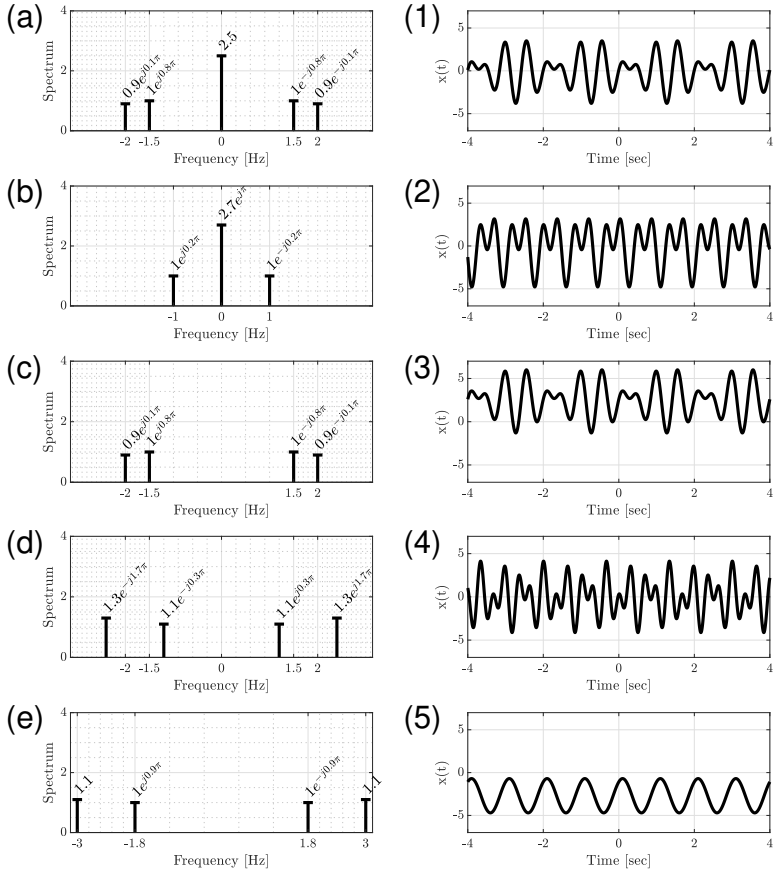

Figure 1: Five signals with their corresponding spectra.

Exercise 3
The frequency spectrum of the signal $x(t)$ is shown in Fig. 2.

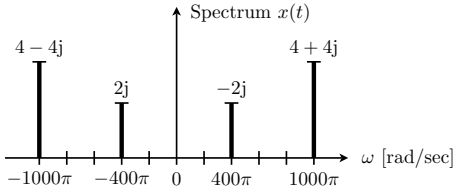

Figure 2: Frequency spectrum of $x(t)$.

Obtain a formula for the signal $x(t)$ as a sum of sinusoidal signals, i.e., in the form

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(\omega_k t + \phi_k).$$

Notes: Make sure that the amplitudes A_k are real-valued. Furthermore note that in Fig. 2 the horizontal axis of the spectral plot denotes the frequency in [rad/sec], with $\omega_k = 2\pi f_k$ and that the values of the bars are not given in Polar notation but in Cartesian notation.

Exercise 4

[P1] The incomplete spectrum of the *real* signal $x(t)$ is shown in Fig. 3.

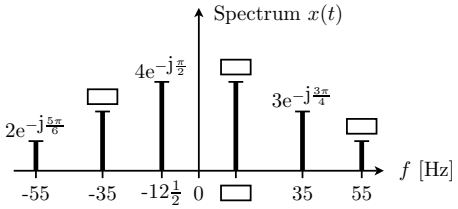

Figure 3: Spectrum of $x(t)$.

a. Fill in the empty boxes for the missing components.

b. Write an equation for $x(t)$ in terms of sinusoidal signals:

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k).$$

Exercise 5

Given the spectrum of signal $x(t)$ in Fig. 4. Draw the spectrum of the following signals.

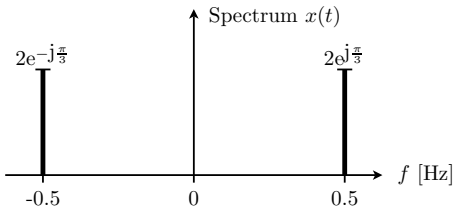

Figure 4: Spectrum of $x(t)$.

Note: Remember to label your axes and indicate the complex amplitudes in polar notation as in Fig. 4. Try to obtain your answers using as few mathematical derivations as possible.

a. $y_1(t) = 3x(t) - 1$,

b. $y_2(t) = x(t-1)$,

c. $y_3(t) = x(t) \cdot \cos(2.4\pi t)$.

Exercise 6

[P2] The signal $x(t)$ is formed from the signal $v(t)$ by amplitude modulation. Assume that

$$v(t) = 3 + 3\cos(10\pi t + \pi/3), \qquad \text{and} \qquad x(t) = v(t) \cdot \cos(40\pi t).$$

a. Draw the spectrum for $v(t)$.

b. Draw the spectrum for $x(t)$.