Answers of Exercises

Module Basics Sampling and Reconstruction

Note:

- The symbol [P] in the margin of an exercise denotes there is a pencast available.

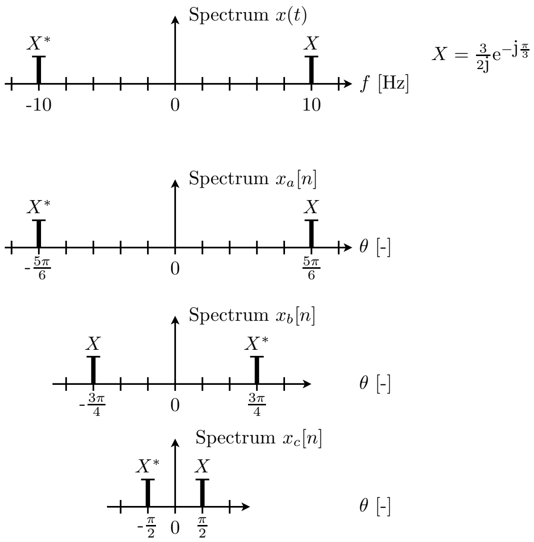
Exercise 1

[P1]

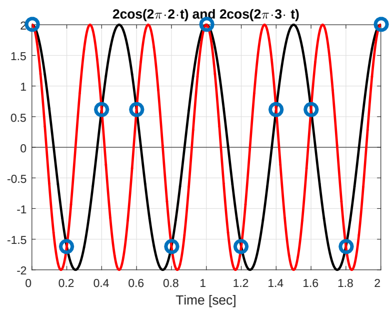
a. $x_a[n] = 3\sin(\frac{5\pi}{6}n - \frac{\pi}{3})$

b. $x_b[n] = 3\sin(\frac{3\pi}{4}n - \frac{2\pi}{3})$

c. $x_c[n] = 3\sin(\frac{\pi}{2}n - \frac{\pi}{3})$



Exercise 2

a. 

b. Equalities thus occur 5 times per second, or at a frequency of 5 [Hz].

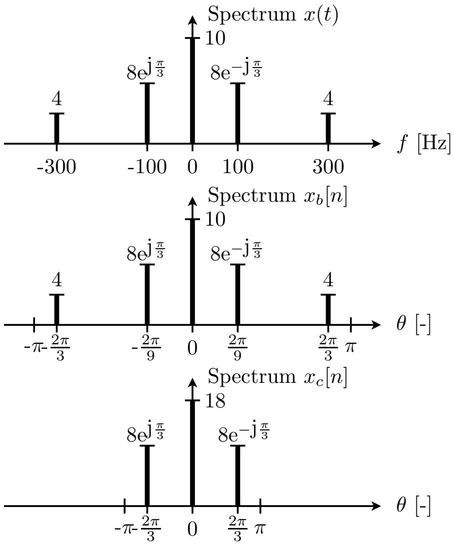
c. $x_1[n] = x_2[n] = 2\cos(0.8\pi n)$

Exercise 3

a. $x(t) = 10 + 16\cos(200\pi t - \frac{\pi}{3}) + 8\cos(600\pi t)$

b. $x[n] = 10 + 16\cos(\frac{2\pi}{9}n - \frac{\pi}{3}) + 8\cos(\frac{2\pi}{3}n)$

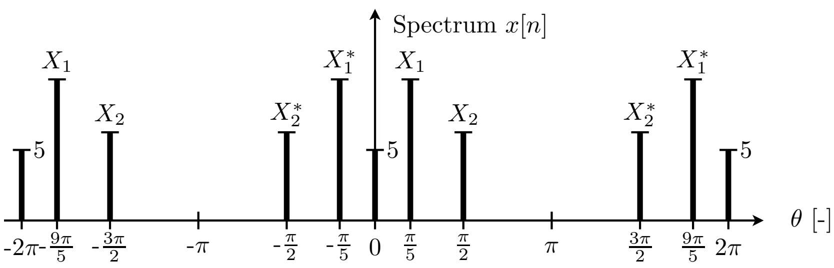
c. $x[n] = 18 + 16\cos(\frac{2\pi}{3}n - \frac{\pi}{3})$



Exercise 4

a. $x(t) = 5 + 20\cos(40\pi t - \pi/4) + 12\cos(100\pi t + \pi/3)$.

$X_1 = 10e^{-j\frac{\pi}{4}} \quad X_2 = 6e^{j\frac{\pi}{3}}$

b. 

Exercise 5

a. Signal is real.

b. $x(t) = 4\cos(80\pi t - \pi/2) + 8\cos(24\pi t - \pi/3)$

c. $x(t)$ is periodic with period $T_0 = \frac{1}{4}$ [sec].

d. $f_s > 80$ [samples/sec].

Exercise 6

a. Signal is periodic with period $T_0 = \frac{1}{200}$ [sec].

b. $x(t) = \frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{3}}\cos(2400\pi t - \frac{\pi}{4}) + \frac{1}{\sqrt{3}}\cos(4800\pi t - \frac{\pi}{3})$

c. $f_s > 4800$ [samples/sec].

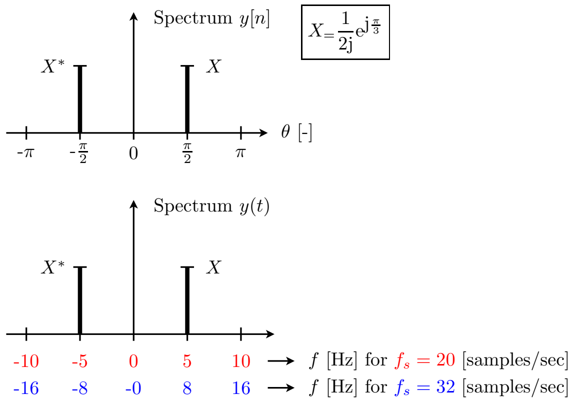
Exercise 7

[P2] First possibility: $x(t) = 10\cos(1000\pi t - \frac{\pi}{3})$
Second possibility: $x(t) = 10\cos(4000\pi t + \frac{\pi}{3})$

Exercise 8

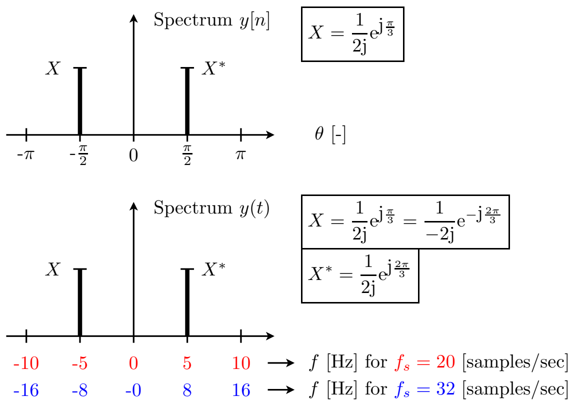
a. $y(t) = \sin(10\pi t + \frac{\pi}{3})$

b. $y(t) = \sin(16\pi t + \frac{\pi}{3})$



c. $y(t) = \sin(10\pi t + \frac{2\pi}{3})$

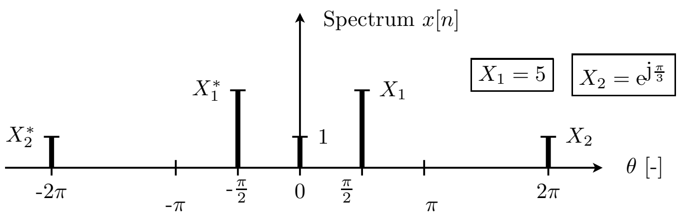
d. $y(t) = \sin(16\pi t + \frac{2\pi}{3})$



Exercise 9

a. $f_s > 40$ [samples/sec].

b. $x[n] = 10\cos(\frac{\pi}{2}n) + 1$



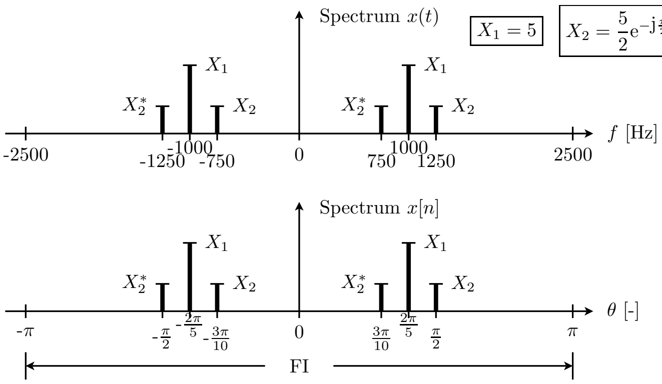
Exercise 10

a.

$$x(t) = X_1 e^{j2000\pi t} + X_1^* e^{-j2000\pi t} + X_2 e^{j2500\pi t} + X_2^* e^{-j2500\pi t} + X_2^* e^{j1500\pi t} + X_2 e^{-j1500\pi t}$$

with $X_1 = 5$ and $X_2 = \frac{5}{2} e^{-j\frac{\pi}{2}}$. This signal is periodic with Fundamental frequency $F_0 = 1/T_0 = 250$ [Hz].

b. $f_s > 2(1250) = 2500$ [samples/sec].



c.

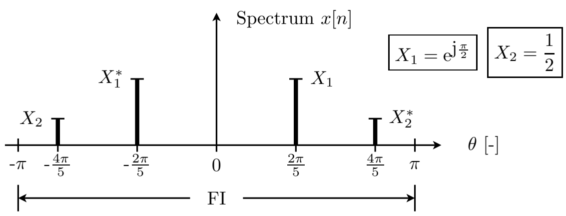
Exercise 11

a. $f_s > 2(150) = 300$ [samples/sec].

b. We can evaluate a mathematical expression to describe the samples $x[n]$ as follows:

$$x[n] = 2\cos(0.4\pi n + \pi/2) + \cos(0.8\pi n).$$

The spectal plot of $x[n]$ is depicted in the figure.



c. $f_s = 150$ [samples/sec].

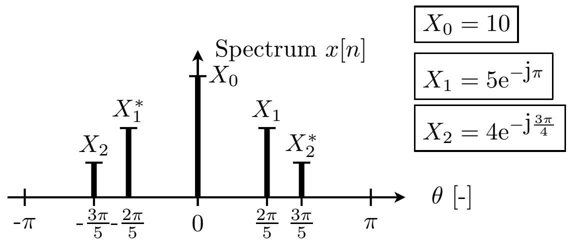
Exercise 12

[P3]

C/D: $f_{si} = 800$ [Hz]
D/C: $f_{so} = 400$ [Hz].

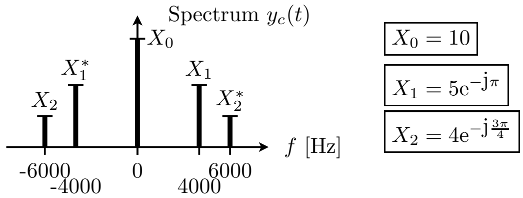
Exercise 13

a.



b.

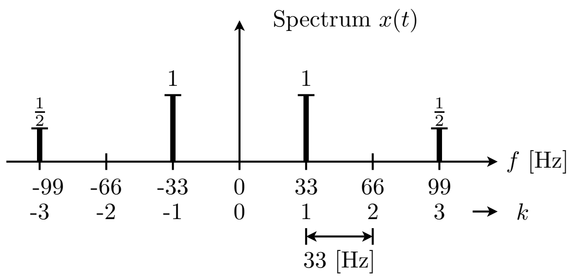
$$y(t) == 10 + 10\cos(4000\pi t - \pi) + 8\cos(6000\pi t + 3\pi/4)$$



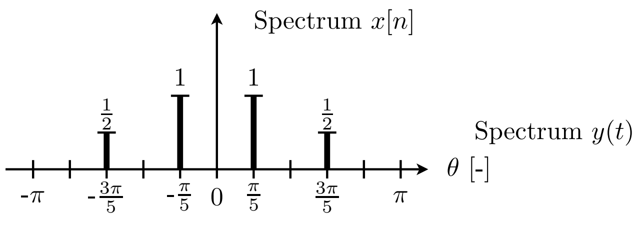
c. $\Rightarrow y_c(t) = 10 + 10\cos(8000\pi t - \pi) + 8\cos(12000\pi t + \frac{3\pi}{4})$

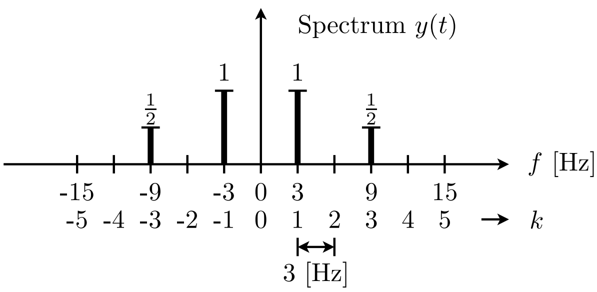
Exercise 14

a. $F_0 = 1/T_0 = 33$ [Hz].



b. $x[n] = x(t)|_{t=n/f_s} = 2\cos(\frac{\pi}{5}n) + \cos(\frac{3}{5}\pi n)$.





c. $y(t) = 2\cos(\frac{\pi}{5}30t) + \cos(\frac{3}{5}\pi 30t) = 2\cos(6\pi t) + \cos(18\pi t)$. $F_0 = 1/T_0 = 3$ [Hz].

d. We observe for $f_s = 30$ [Hz] that $x(t)$ has a fundamental frequency of 33 [Hz] and $y(t)$ has a fundamental frequency of 3 [Hz] - a eleven fold difference in frequency. This illustrates that the fundamental frequency can be changed by altering the sampling rate of a sampling system.